ANDERSON

Characteristics of Steel Wire

for Transmission Lines

Electrical Engineering

M. S.

1912

CHARACTERISTICS OF STEEL WIRE FOR TRANSMISSION LINES

BY

CHARLES THOMAS ANDERSON

B. S., University of Illinois, 1911

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

IN ELECTRICAL ENGINEERING

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1 9 1 2

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 31, 1912

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

CHARLES THOMAS ANDERSON

ENTITLED CHARACTERISTICS OF STEEL WIRE FOR TRANSMISSION LINES

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Ernst Berg In Charge of Major Work

Ernst Berg Head of Department

Recommendation concurred in:

Morgan Brooks
Ellery B. Paine
J M Bryant

Committee on Final Examination

CONTENTS

THE CHARACTERISTICS OF STEEL WIRE FOR TRANSMISSION LINES

I INTORDUCTION

In the transmission of electrical energy the material of the conductor has been, until quite recently, almost exclusively copper. But with the introduction of high voltage transmission, the line conductivity has come to be of less importance than before, and in lines carrying small power, of less importance than the mechanical strength of the wire. Hence there arises inquiry as to the practicability of steel wire as a conductor in electrical transmission.

Since practically all high voltage power is alternating, it is pertinent to investigate the characteristics of steel wire as a conductor of alternating currents.

Alternating current in passing through a conductor distributes itself most densely near the surface. In extreme cases the resistance loss and voltage drop are large as compared to those due to direct current. In copper except for very large conductors and very high frequencies this effect is negligible; but in steel it is quite appreciable even for small conductors and low frequencies. This phenomenon which is known as "skin effect" is the most important problem in connection with the characteristics of steel wire in alternating power transmission.

BIBLIOGRAPHY

"Absolute Measurements in Electricity and Magnetism" A. Gray.

"Alternating Currents" Russel.

"Elements of Electricity and Magnetism" J. J. Thomson.

"Recent Researches in Electricity and Magnetism", J. J. Thomson.

"Principles of Electrical Engineering" Pender.

"Transient Electric Phenomena and Oscillations" Steinmetz.

"Electric Discharges Waves and Impulses" Steinmetz

Bureau of Standards Bulletin Vol.5, No. 1.

II THEORY

If an alternating voltage is impressed upon a circuit containing ohmic resistance and self-inductive reactance, there is a flow of electrical energy or a current. The magnitude of this current and its phase relation to the impressed voltage are determined by the resistance and reactance.

$$I = E/(r - jx) \; ; \quad \alpha = \tan^{-1} x/r .$$

The value of (X) depends upon the self inductance of the circuit and the frequency

$$x = 2\pi f L \cdot 10^{-8}$$

where (f) is the frequency of the alternating voltage and the inductance (L) is the number of interlinkages of the conductor with magnetic lines at unity.

If now the conductor be considered built up of concentric tubular elements and an alternating voltage be impressed upon all of these elements, whatever the distribution of current among the tubes, any given element will be interlinked with all of the flux of currents in tubes exterior to it. But it will be interlinked with only that part of the flux of the currents interior to it which are set up outside of it. The innermost element will be interlinked with all of the flux, as one extreme, whereas the outermost element will be interlinked with none of the internal flux. Consequently there will be a gradual increase of self-inductive reactance and of current lag (α) and a gradual decrease of current magnitude as the center of the conductor is approached.

It is noteworthy that the greater the internal flux in pro-

portion to the total flux, the more marked will be the unequal current distribution. Hence, since a high magnetic permeability permits a high value of flux density, considering a given value of magnetomotive-force acting in a material, it is evident that this phenomenon is pronounced in a steel conductor.

In connection with this problem a mathematical treatment of alternating current distribution, given by Steinmetz in his "Transient Electric Phenomena and Oscillations" has been carefully reviewed. It is fitting, herein, to give a digest of this treatment including such mathematical steps as seem convenient.

Let figure (1) Plate I represent a cross section of a flat conductor, the electric conductivity and magnetic permeability of which are (λ) and (μ) respectively. Let (l) be the distance measured from the center; and let ($2l_0$) be the total thickness of conductor.

Let (E_1) be the total supply voltage per unit length of conductor; let (E_2) be the external reactance voltage (due to flux external to conductor) per unit length of conductor; and let

$$E_0 = E_1 - E_2,$$

be the voltage consumed per unit length inside the conductor.

Let $I = i_1 + j i_2$, be the current density, $\mathfrak{B} = b_1 + j b_2$, the magnetic flux density at (dl); and let (E) be the electro-motive-force consumed per unit length, at the conductor element (dl) by self-inductive reactance due to the internal magnetic field.

Then $I\,dl$ in the element represents the field intensity

$$d\mathcal{H} = 4\pi I\,dl$$

which increases the magnetic density from the inner to the outer edge of the element (dl) by

$$d\mathcal{B} = \mu\,d\mathcal{H}$$

$$= 4\pi\mu I\,dl. \tag{2}$$

The difference of flux $(\mathcal{B}\,dl)$ from the inner side to the outer side of (dl) causes the electro-motive-force consumed self-inductive reactance to be greater at the inner edge than at the outer edge of the element (dl) by

$$dE = +j\,2\pi f\mathcal{B}\,dl\,10^{-8} \tag{3}$$

because the inner edge is interlinked with $(\mathcal{B}\,dl)$ more flux than is the outer edge. That is although the flux density decreases, the voltage consumed by self-inductive reactance increases as the center of the conductor is approached.

E includes the e.m.f.,(E) consumed by internal self-inductive reactance and the e.m.f., $\frac{I}{\lambda}$ consumed by ohmic resistance of the element per unit length of conductor, or

$$E_0 = E + \frac{I}{\lambda}. \tag{4}$$

Differentiating,

$$dE = -\frac{1}{\lambda}\,dI. \tag{5}$$

Substitute (5) in (3)

$$dI = -2j\pi f\mathcal{B}\,10^{-8}\,dl\,\lambda.$$

$$\frac{d^2I}{dl^2} = -2j\pi f\lambda\,10^{-8}\frac{d\mathcal{B}}{dl}.$$

Substitute value $\frac{d\mathcal{B}}{dl}$, (2):

$$\frac{d^2I}{dl^2} = -8j\pi^2\lambda\mu f I\,10^{-8}.$$

Let $C^2 = a^2 f = 0.4\,\pi^2 \lambda \mu f\, 10^{-8}$

$$a^2 = 0.4\,\pi^2 \lambda \mu\, 10^{-8}$$

$$\frac{d^2 I}{d l^2} = -2 j c^2 I.$$

Given the form

$$\frac{d^2 y}{d x^2} + c \frac{d y}{d x} + a y = b$$

$$y = A_1 \varepsilon^{+K_1 x} + A_2 \varepsilon^{K_2 x} + \frac{b}{a}.$$

and solving for K,

$$m^2 + c m + a = 0$$

$$m = -\frac{c}{2} \pm \sqrt{\frac{c^2}{4} - a}.$$

$$\frac{d^2 I}{d l^2} + 2 j c^2 I = 0.$$

$$m^2 + 2 j c^2 = 0.$$

$$m = \sqrt{-2 j c^2} = \pm c(1-j)$$

$$I = A_1 \varepsilon^{c(1-j)l} + A_2 \varepsilon^{-c(1-j)l}$$

But since I is the same for $(+l)$ and $(-l)$

$$A_1 = A_2 = A.$$

$$\therefore I = A\left(\varepsilon^{+c(1-j)l} + \varepsilon^{-c(1-j)l}\right)$$

$$\varepsilon^{\pm c(1-j)l} = \cos cl \pm j \sin cl \qquad (16)$$

$$\varepsilon^{+c(1-j)l} = \varepsilon^{cl - jcl} = \varepsilon^{cl}(\cos cl - j \sin cl)$$

$$\varepsilon^{-c(1-j)l} = \varepsilon^{-cl + jcl} = \varepsilon^{-cl}(\cos cl + j \sin cl)$$

$$\varepsilon^{+c(1-j)l} + \varepsilon^{-c(1-j)l} = \left(\varepsilon^{+cl} + \varepsilon^{-cl}\right)\cos cl - j\left(\varepsilon^{+cl} - \varepsilon^{-cl}\right)\sin cl.$$

$$I = A\left[\left(\varepsilon^{+cl} + \varepsilon^{-cl}\right)\cos cl - j\left(\varepsilon^{+cl} - \varepsilon^{-cl}\right)\sin cl\right].$$

To solve for the constant (A), when $l = l_0$ or at conductor surface

$$I_1 = A\left[\left(\varepsilon^{+cl_0} + \varepsilon^{-cl_0}\right)\cos cl_0 - j\left(\varepsilon^{+cl_0} - \varepsilon^{-cl_0}\right)\sin cl_0\right].$$

But at the surface there is no e.m.f. of self inductance due to internal field.

$$I_0 = \lambda E_0$$

Hence $I_0 = A\,[(\varepsilon^{+cl_0} + \varepsilon^{-cl_0}) \cos cl_0 - j(\varepsilon^{+cl_0} - \varepsilon^{-cl_0}) \sin cl_0]$.

$$A = \frac{\lambda E_0}{[(\varepsilon^{+cl_0} + \varepsilon^{-cl_0}) \cos cl_0 - j(\varepsilon^{+cl_0} - \varepsilon^{-cl_0}) \sin cl_0]}$$

$$\therefore I = \lambda E_0 \frac{(\varepsilon^{+cl} + \varepsilon^{-cl}) \cos cl - j(\varepsilon^{+cl} - \varepsilon^{-cl}) \sin cl}{(\varepsilon^{+cl_0} + \varepsilon^{-cl_0}) \cos cl_0 - j(\varepsilon^{+cl_0} - \varepsilon^{-cl_0}) \sin cl_0} \quad (20)$$

$$(\varepsilon^{+cl} + \varepsilon^{-cl}) \cos cl - j(\varepsilon^{+cl} - \varepsilon^{-cl}) \sin cl$$

$$= \sqrt{(\varepsilon^{+cl} + \varepsilon^{-cl})^2 \cos^2 cl + (\varepsilon^{+cl} - \varepsilon^{-cl})^2 \sin^2 cl}$$

$$= \sqrt{(\varepsilon^{2cl} + \varepsilon^{-2cl} + 2) \cos^2 cl + (\varepsilon^{2cl} + \varepsilon^{-2cl} - 2) \sin^2 cl}$$

$$= \sqrt{(\varepsilon^{2cl} + \varepsilon^{-2cl}) + 2 \cos 2cl}\,.$$

In like manner,

$$(\varepsilon^{cl_0} + \varepsilon^{-cl_0}) \cos cl_0 - j(\varepsilon^{cl_0} - \varepsilon^{-cl_0}) \sin cl_0$$

$$= \sqrt{\varepsilon^{2cl_0} + \varepsilon^{-2cl_0} + 2 \cos 2cl_0}\,.$$

$$\therefore I = \lambda E \sqrt{\frac{\varepsilon^{2cl} + \varepsilon^{-2cl} + 2 \cos 2cl}{\varepsilon^{2cl_0} + \varepsilon^{-2cl_0} + 2 \cos 2cl_0}}\,.$$

The current density at the center $(l = 0)$, by substituting in (20),

$$I_0 = \frac{2 \lambda E_0}{(\varepsilon^{+cl_0} + \varepsilon^{-cl_0}) \cos cl_0 - j(\varepsilon^{+cl_0} - \varepsilon^{-cl_0}) \sin cl_0}$$

$$= \frac{2 \lambda E_0}{\sqrt{\varepsilon^{+2cl_0} + \varepsilon^{-2cl_0} + 2 \cos 2cl_0}}\,.$$

The mean value of current density through the conductor is

$$I_m = \frac{1}{l_0} \int_0^{l_0} I \, dl\,.$$

$$I = A\left(\varepsilon^{+c(1-j)l} + \varepsilon^{-c(1-j)l}\right).$$

$$I_m = \frac{1}{l_0}\int_0^{l_0} I\,dl = \frac{A}{(1-j)cl_0}\left[\varepsilon^{(1-j)cl} - \varepsilon^{-(1-j)cl}\right]_0^{l_0}$$

$$= \frac{A\left(\varepsilon^{(1-j)cl_0} - \varepsilon^{-(1-j)cl_0}\right)}{(1-j)cl_0}.$$

Substituting (16) and (A)

$$I_m = \frac{\lambda E_0\left[(\varepsilon^{cl_0} - \varepsilon^{-cl_0})\cos cl_0 - j(\varepsilon^{cl_0} + \varepsilon^{-cl_0})\sin cl_0\right]}{(1-j)cl_0\left[(\varepsilon^{cl_0} + \varepsilon^{-cl_0})\cos cl_0 - j(\varepsilon^{cl_0} - \varepsilon^{-cl_0})\sin cl_0\right]}$$

$$= \frac{\lambda E}{cl_0\sqrt{2}}\sqrt{\frac{\varepsilon^{+2cl_0} + \varepsilon^{-2cl_0} - 2\cos 2cl_0}{\varepsilon^{+2cl_0} + \varepsilon^{-2cl_0} + 2\cos 2cl_0}}.$$

Therefore the increase of effective impedance of interior of wire over ohmic resistance (R_0) is

$$\frac{Z}{R_0} = \frac{I_0}{I_m}$$

$$= \frac{1}{(1-j)cl_0}\,\frac{(\varepsilon^{+cl_0} + \varepsilon^{-cl_0})\cos cl_0 - j(\varepsilon^{+cl_0} - \varepsilon^{-cl_0})\sin cl_0}{(\varepsilon^{+cl_0} - \varepsilon^{-cl_0})\cos cl_0 - j(\varepsilon^{+cl_0} + \varepsilon^{-cl_0})\sin cl_0}. \quad (28)$$

$$= \frac{1}{cl_0\sqrt{2}}\sqrt{\frac{\varepsilon^{+2cl_0} + \varepsilon^{-2cl_0} + 2\cos 2cl_0}{\varepsilon^{+2cl_0} + \varepsilon^{-2cl_0} - 2\cos 2cl_0}}. \quad (29)$$

Now if the conductor is large enough that ε^{-cl_0} can be neglected in comparison to ε^{+cl_0} then I in the center of the conductor is negligible and when (l) approaches (l_0) from (20) we have

$$I = \lambda E_0\,\frac{\varepsilon^{+cl}(\cos cl - j\sin cl)}{\varepsilon^{+cl_0}(\cos cl_0 - j\sin cl_0)}$$

mult. both sides by $(\cos cl_0 + j\sin cl_0)$

$$I = \lambda E_o \mathcal{E}^{c(l-l_o)}[\cos c(l-l_o) - j \sin c(l-l_o)$$

Let $l_o - l = s$:

$$I = \lambda E_o \mathcal{E}^{-cs}(\cos cs + j \sin cs)$$

$$= \lambda E_o \mathcal{E}^{-cs}.$$

Neglecting $\mathcal{E}^{-2cl_o}$ and noting that $\cos 2cl_o$ becomes small when $\mathcal{E}^{+2cl_o}$ becomes large.

$$I_m = \frac{\lambda E_o}{\sqrt{2}\, c l_o}.$$

Current density in absence of skin effect or at surface is

$$I_o = \lambda E_o.$$

$$\frac{Z}{R_o} = \frac{I_o}{I_m} = (1-j)\, c l_o,$$

$$= c l_o - j c l_o$$

$$= c l_o \sqrt{2}.$$

Therefore, for very thich conductors the general equation (28) + (29) reduces to the above and

$$Z = R_o (c l_o - j c l_o)$$

$$= R' - j X'.$$

$$R' = X' = R_o c l_o = \frac{\pi \sqrt{4 \mu \lambda f}}{10^4} = \frac{\sqrt{\mu \lambda f}}{5030}.$$

That is, in the extreme case where the penetration is slight the reactance and resistance are equal.

It is seen that only $\frac{1}{cl_0}$ of the conductor section is effective so that the depth of the penetration is

$$l_p = \frac{l_0}{cl_0} = \frac{1}{c}.$$

considering the current density constant down thru the layer $\frac{1}{c}$.

$$c^2 = \alpha^2 f$$
$$= 0.4\,\pi^2 \lambda \mu f\, 10^{-8}.$$

$$l_p = \frac{10^4}{\pi\sqrt{0.4\lambda\mu f}} = \frac{5030}{\sqrt{\lambda\mu f}}.$$

From these equation it appears that the effective resistance and effective internal reactance vary directly as the square root of the permeability and the conductivity of the conductor material and the frequency and that the depth of penetration of the current into the conductor varies inversely as the square root of these factors.

The internal inductance, however, decreases with the crowdind of the current toward the surface of the conductor and reaches zero in the limit.

In the above discussion hysteresis loss in magnetic material has not been considered. In the case of low frequencies and small steel conductors in which the penetration of the current is considerable, there is a hysteresis loss of importance in transmission line calculation. The loss due to hysteresis is

$$W = \eta \mathcal{B}^{1.6},$$

where (W) is ergs per cycle per cubic centimeter of the material in which the changing flux is considered; and where (η) is a constant for the material considered; and where ($\mathcal{B}$) is the magnetic flux density in lines per square centimeter where the

loss takes place.

In a different form we may write

$$P = 2 f B^{1.6} 10^{-10}$$

where (P) is watts and (f) is frequency in cycles per second.

If the effective resistance of a steel transmission line is calculated by dividing the line loss in watts by the square of the current causing this loss the value of (R) will be greater than the "skin effect" value because of hysteresis loss. This causes a complication in this method of handling the problem. This point is more fully discussed in section IV.

The following theoretical formula for effective resistance given by Gray in his "Absolute Measurements in Electricity and Magnetism" has been chosen as a check on the results discussed in section IV.

$$R' = R_o \left(1 + \frac{1}{12} \frac{\mu^2 l^2 (2\pi f)^2}{R_o^2} - \frac{1}{180} \frac{\mu^4 l^4 (2\pi f)^4}{R_o^4} + \text{-----} \right)$$

where R' is the effective resistance due to inequal current distribution and (R) is the ohmic resistance of the length (l) of conductor, and where (f) is the frequency.

In some inductance calculations in section IV, the following formula taken from the Bureau of Standards bulletin Vol. 5 #1, was used

$$L = 4 l \left(\log_\varepsilon \frac{D}{\rho} + \frac{\mu}{4} - \frac{D}{l} \right) 10^{-9} \qquad (79)$$

where (L) is the inductance in henries (l) is the length of one conductoor of the double line in centimeters; where (D) is the distance between the conductors in centimeters and (ρ) the radius of the wire in the same units; and where (μ) is the magnetic permeability of the line material.

This formula of inductance allows the following separation of the internal inductive reactance from the total reactance (X_T)

$$X_T = 2\pi f L_T$$

$$L_T = 4l\left(\log_\varepsilon \frac{D}{\rho} + \int_{\rho_p}^{\rho_o} \frac{x^3 dx}{\rho_o^4} - \frac{D}{l}\right) 10^{-9}$$

$$X_{int.} = 8\pi f l\left(\int_{\rho_p}^{\rho_o} \frac{\mu x^3 dx}{\rho_o^4}\right) 10^{-9}$$

$$= 8\pi f l\left(\frac{\mu}{4}\right), \text{ if } \rho_p = 0, \text{ or } l_p = \rho_o$$

$$X_{air} = 8\pi f l\left(\log_\varepsilon \frac{D}{\rho} - \frac{D}{l}\right) 10^{-9}.$$

The formula given by Gray for skin effect inductance is

$$L' = l\left[A + \mu\left(\frac{1}{2} - \frac{1}{48}\frac{\mu^2 l^2 (2\pi f)^2}{R^2} + \frac{13}{8640}\frac{\mu^4 l^4 (2\pi f)^4}{R^4}\right)\right]$$

where (A) is a constant.

PLATE I.

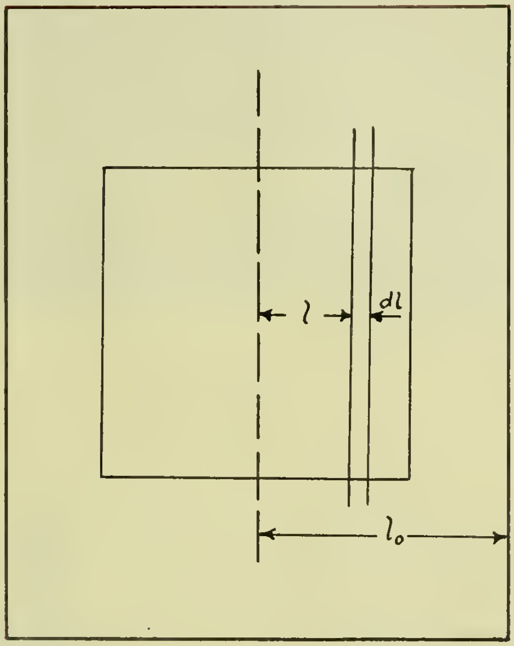
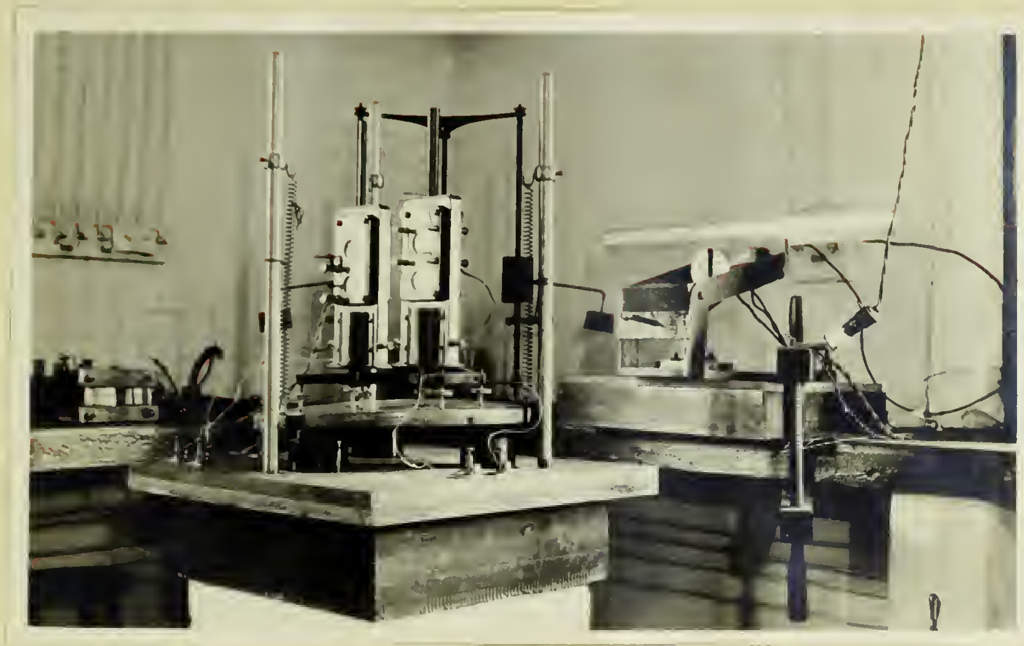
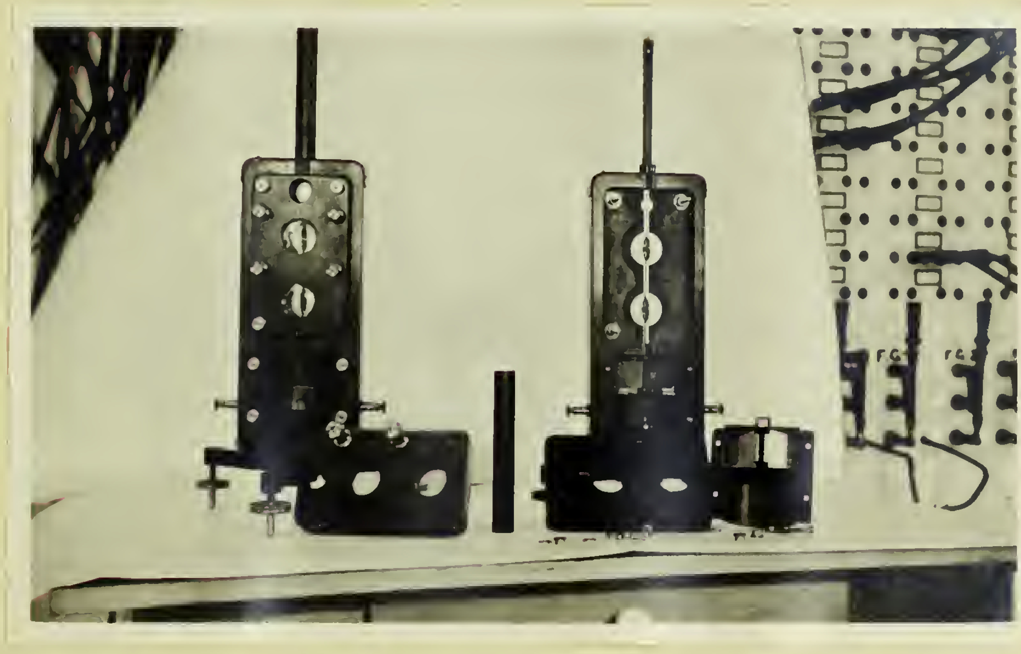


Fig. 1.

PLATE II.





III MANIPULATION.

It was desired to test for the relation of effective resistance and internal reactance to current density and the frequency of the current in a steel conductor. The method of obtaining data pertaining to these relations was to measure voltage drop and power loss in the conductor carrying the current, for various current densities with constant frequency and for various frequencies with a constant current density. This method was based on the following relations:

$$Z = \frac{E}{I} \; ; \quad R = \frac{W}{I^2} \; ;$$

$$X_T = \sqrt{Z^2 - R^2}$$

$$X_{int} = X_t - X_{air} .$$

where (Z) is the total impedance; (E) the voltage drop over the conductor; (I) is the current in the conductor; (W) is the power loss, as measured, in the conductor; (R) is the resistance of conductor which would cause this power loss at the given current (I); (X_T) is the total reactance;(X)int.) is the reactance due to flux inside the conductor; and (Xair) that part of the total reactance which is due to the flux in the air between the two sides of the parallel return conductor.

The first conductor tested was a stranded, galvanized steel cable made up of seven strands each of which was one eighth inch in diameter, and sixty-three feet, ten inches in length. This was suspended along a floor beam as a parallel return conductor loop, the distance between sides being six inches.

The impressed voltage necessary to produce the maximum allowable current in this conductor was about two volts whereas

the currents desirable for the tests ranged from zero to thirty amperes. It is evident that this combination of voltage and current values is difficult to handle with commercial wattmeters and voltmeters.

As it was desired to make a preliminary test with rough commercial instruments, it was necessary because of these extremes of current and voltage magnitudes to build a suitable potential transformer. With the voltage impressed upon the meters thus raised by a known ratio from the actual impressed voltage, the commercial wattmeter and voltmeter were used with some degree of satisfaction. The connections used are shown digrammatically in PlateIII figure 2.

But this ratio of apparent to actual voltage was not so easily calculated as might be expected. For although the resistance drop in the potential transformer could be calculated with reasonable accuracy, the reactance drop and the change of phase relation with the line current, caused by the transformer reactance, between the real and the apparent voltage were uncertain quantities, since the reactance could be only roughly estimated.

The results of this test were therefore considered only as rough estimates of the magnitudes to be dealt in subsequent tests; and the results are not presented here.

In the final tests were used a voltmeter dynamometer and wattmeter dynamometer manufactured by the General Electric Company. Photographs of these instruments are included (Plate II) to afford a general idea of their structure. Their heighth is about twenty inches. These meters are worthy of description.

Each dynamometer consists of a vertical suspension carrying two moving coils, one above the other, and two pairs of stationary coils, each pair enclosing and at right angles to a moving coil. The suspension is six inches of phosphor-bronze wire, .002 inches in diameter. This is fastened at the top of the suspension tube. The suspension carries a small mirror for reflecting a beam of light, the two movable coils and a four-vaned damping fan, all connected rigidly together. Below these, connection is made with a terminal through a helical coil of the .002 inch phosphor-bronze wire. In the walls enclosing the suspended parts are three small windows of mica which make it possible to see the moving coils and the position of the axis of suspension with reference to the frame of the instrument, and a fourth window containing a lens of one meter focal length for the passage of the beam of light to the mirror and back to the measuring screen. The instrument is mounted upon a tripod with thum-screws for leveling.

The stationary coils of the wattmeter are used, as current coils, in series with a load to be measured. The movable coils are used as the potential circuit, being connected in shunt with the load to be measured. The sensibility of this instrument is a deflection of 500 millimeters on a screen one meter from the mirror for a power measurement of 0.128 watts.

The six coils of the volt meter are connected in series and shunted across the circuit whose voltage is to be measured. The sensibility of this instrument is a reflection of 500 millimeters on a scale at one meter distance from the mirror for 1.08 volts.

For varying the sensibility of these instruments, resistance from (0) to 10,000 ohms can be connected in series with the potential circuits.

Some of the difficulties of manipulating these dynamometers may well be discussed before taking up the description of the tests.

It was found that the vibrations from running machinery in the building were transmitted from the foundation of the building and through the piers upon which the instruments were mounted. These vibrations set up oscillations in the suspension which forbade any careful measurements. To surmount this difficulty it was necessary to build a platform carrying three upright standards and a second platform suspended from the standards by coil springs. On the lower side of the suspended platform were located three plungers which were immersed in cups of heavy oil. This precaution was taken to insure the meters against such slight disturbances as passing drafts of air, without, at the same time, furnishing a medium for the passage of the vibrations from the pier.

When the meters were mounted upon this suspended platform, they were found to vibrate somewhat because the suspending springs picked out and responded to certain frequencies of vibration. This evil was practically eradicated by stretching rubber bands from coil to coil of the springs until the proper arrangement was found.

Even after these precautions slight breezes were found to be serious enough in their results that shields of paper about the standards were necessary.

It must be said that this form of "set up" was very unsatisfactory; because every time the meters were used it was necessary to readjust their balance. This trouble, however, was probably due to the warping of the suspended platform. But with such an arrangement the meters are liable to a great many injurious disturbances.

There is another difficulty in connection with the dynamometers which is difficult to remedy. Slight changes of temperature of the helical coil at the base of the suspended parts cause not only a shifting of the equililbrium position of the mirror but cause a variation of the angle of deflection for given values of torgue. This threw error into all of the results which limited their accuracy to within about two per cent. The temperature changes in the spiral were due partly to the current flowing through it and partly to changes in room temperature. The former trouble was in great degree avoided by properly observing the zero reading. But the former cause made it necessary to take each series of readings within the shortest possible time to avoid serious breaks in the curves plotted from the data.

After the dynamometers were properly set up they were calibrated with secondary standards according to the desired range of magnitudes.

In the tests it was found necessary to use the dynamometers separately because the magnetic field from the wattmeter caused serious deflections of the voltmeter movable coil when the two meters were in simultaneous operation. Consequently it was desirable to run current-voltage tests

separately from current-power tests.

The complete set-up for the tests can best be understood with the help of the diagram (3) of plate II.

A General Electric alternating current Generator (G) of 7.5 K. W. capacity, 220 volts, 1200 revolutions per minute and of type A.H.B. was driven by belt from a Crocker Wheeler Direct current motor (M) of 10 H.P. capacity, 230 volts. A variable speed over a wide range was obtained by the use of 110 and 220 volts for the motor armature operation, using an armature rheostat for the lowest speeds, also by variations in the motor excitation. With the variation the frequency of the voltage from the alternator was varied from 23 to 90 cycles per second. The alternating voltage thus developed was "steped up" through a General Electric transformer of 7.5 K. W. capacity and 110-220 volts (T). The resistances (R) was used only in starting. This voltage was then impressed upon a 500 W. transformer having a ratio of about 50. to 1. through a potentiometer arrangement of rheostats (R) and (R) and thus decreased at (T) to the low value required by the low resistance circuit of the cable (C) and wattmeter (W) current coil, and ammeters (A) and (A). This circuit, as can be seen by the diagram, included the cable, the ammeter (A) and the wattmeter current coil in series, a shunt over the wattmeter branch at (A) for the purpose of allowing current to flow between readings to hold the temperature of the cable constant, and finally the shunt (S) for shunting out the wattmeter while the voltmeter was in use. Across the cable (C) at (V) was connected the voltage circuit leading to the volt-

age coil of the wattmeter and to the voltmeter. Finally the frequency meter (F) across the generator terminals was of the vibrating reed type.

Upon the cable were six thermometers, imbedded in putty and taped to the wire for the purpose of allowing observations of the temperature. As before mentioned the temperature was held as near constant as possible (within 3 degrees, centigrade) by passing a current through the cable between readings.

With this set up the cable current was read by the commercial ammeter, (A); the voltage drop over the cable was read by the voltmeter dynamometer (V); and the power loss in the cable was read by the wattmeter-dynamometer (W). Seven sets of readings were taken, each including a series of power readings and a series of voltage readings, taken separately. They are the following:

3/8 Inch, stranded, Galvanized steel cable:
60 cycles per second, variable current;
40 " " " " " ;
25 " " " " " ;
20 amperes, variable frequency.
3/16 Inch Solid Steel Cable:
60 cycles per second, variable current;
40 " " " " " ;
25 " " " " " .

PLATE III.

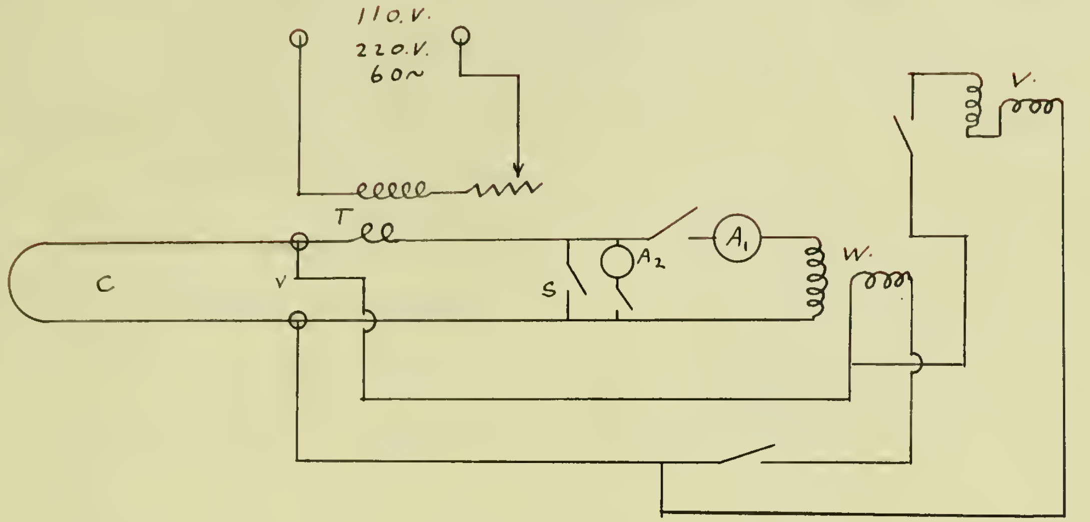


Fig. 2.

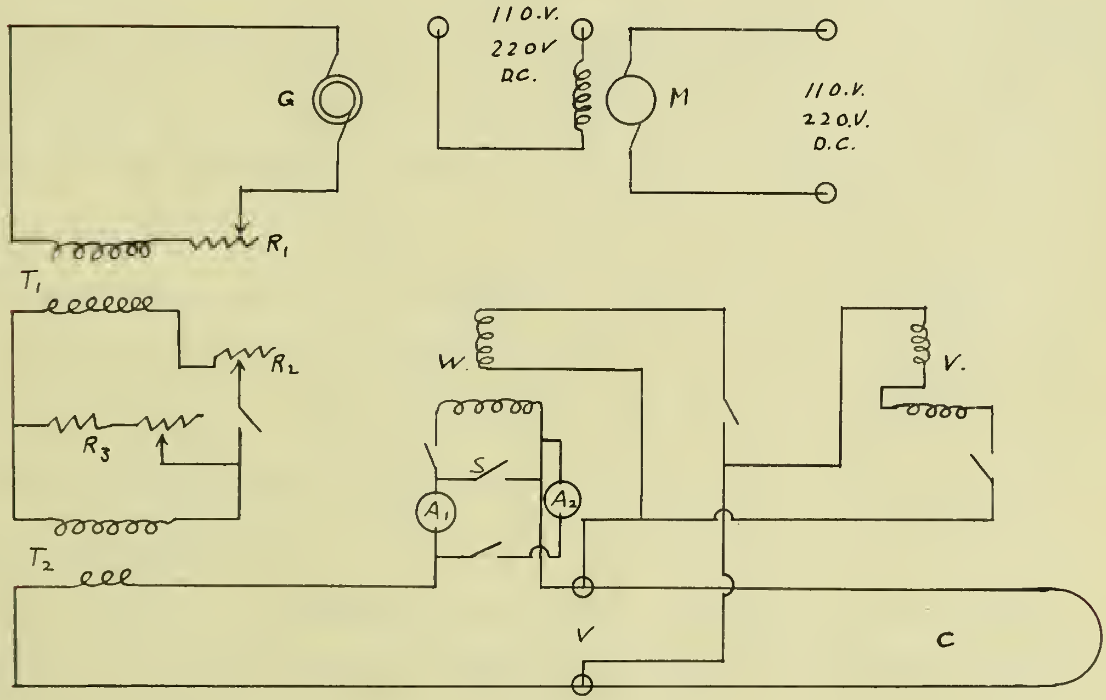


Fig. 3.

IV. EXPERIMENTAL RESULTS AND CONCLUSIONS.

The results of the experimental tests are embodied in the curves on the seven curve-sheets, pages (31) to (44), and the two tables of transmission line constants, pages (45) and (46). This problem of alternating current distribution in a steel conductor, however, should be quite as much one of physics as of engineering. It is therefore desirable not only to point out what phenomena occur but to offer hypotheses in regard to their causes and to speculate on the possibility of further experimental and theoretical investigation. Such is the purpose of this part of the report.

But preliminary to this discussion, a brief account of the methods of deriving the data presented may be of value. The experimental values of (Z), (R) and (W) were plotted. New values of (Z) and (R) were taken from the smooth curves plotted through the experimental data, and were tabulated as (Z_c) and (R_c). The values of (X) calculated from (Z_c) and (R_c) were tabulated and plotted as total reactance. By way of comparison with the total reactance, the values of reactance, for a similar circuit of copper, calculated by the formula given in (II) were plotted for 25 and 60 cycles. Also the values of ohmic resistance as measured by a wheatstone bridge were plotted beside the effective resistance curves.

The tables of transmission line constants for 3/8 inch stranded, galvanized steel cable for 25 and 60 cycles afford an idea of the characteristics of such a line. The items of these tables were calculated from values taken from the curves of (R) and (Z). The calculations were possible for various distances

between conductors, other than the one for which the experimental data were taken, since neither the effective resistance nor the internal reactance involve this distance. Therefore by separating the internal reactances from the total values, as plotted, according to the relations,

$$X_T = 8\pi f l \left(\log_\varepsilon \frac{D}{\rho} + \int_{\rho_p}^{\rho_0} \frac{\mu x^3 dx}{\rho_0^4} - \frac{D}{l}\right)10^{-9};$$

$$X_{int} = 8\pi f l \int_{\rho_p}^{\rho_0} \frac{\mu x^3 dx}{\rho_0^4} 10^{-9};$$

and by adding to the internal reactances, the air reactances calculated with the distances between conductors according to the formula,

$$X_{air} = 8\pi f l \left(\log_\varepsilon \frac{D}{\rho} - \frac{D}{l}\right)10^{-9}$$

the total reactances for the various distances were derived. These tables consist of effective resistance and reactance for six spacing distances and for six values of current density.

The physics of the curves will first be discussed. Then those points which are of especial interest from the standpoint of engineering will be discussed.

Upon referring to curve sheet, page (32) it will be noticed that the effective resistance increases gradually from ohmic, by a law which appears to be,

$$R = K\sqrt{\mu},$$

if we consider the (μ) curve a straight line for some distance, as the approximation on curve sheet page (32) indicates. But when (μ) ceases to follow the constant ratio of increase and the (μ) curve slopes toward the horizontal with the magnetic saturation of the iron, then also the effective resistance curve falls short of the square root law and bends toward the horizontal. The effective resistance curve coincides with the ohmic resistance at very low current densities because the value of

(μ) is so small that the unequal current is so slight as to produce negligible increase of effective over ohmic resistance.

Let it next be noticed that the reactance curve follows a somewhat similar path although the effect of (μ) is more marked. But the impedance curve follows more nearly the form of the effective resistance curve than that of the reactance curve because the impedance is a mere combination of resistance and reactance, and the resistance values for this case are much greater in actual ohms than the reactance values.

But the question arises at once: Why does (μ) increase the values of reactance for low current densities whereas the effect on the effective resistance over the same range is negligible? This question can be answered by an example: It will consist in solving for the value of (μ) necessary to produce the increase in (X) at zero current and substituting this value in a theoretical effective resistance equation to see if the effect upon (R) is appreciable.

$$X_T = 2\pi f L_T = .0054 \qquad f = 60\sim$$

considering $\mu = 1.$

But the value at (0) current on reactance curve page (32) is

$X = .0125\ \omega$ This is due to (μ)

solving for (μ) in the above equation

$$\mu = 20.4\ .$$

Gray's formula for effective resistance (R') is

$$R' = R_o \left(1 + \frac{1}{12}\frac{\mu^2 l^2 (2\pi f)^2}{R_o^2} - \frac{1}{180}\frac{\mu^4 l^4 (2\pi f)^4}{R_o^4} + \cdots\right)$$

$$R_o = \frac{\rho l}{\text{area}} = \frac{1950 \text{ cm.} \cdot 10^4}{0.555 \text{ sq. cm.}} = 3520.10^4 \text{ e.g.s. units}$$

$$R_o^2 = 12.4 \cdot 10^{14}.$$

$$\mu^2 = (20.4)^2 = 417.$$

$$(2\pi f)^2 = (2\pi 60)^2 = 142000,$$

$$\frac{1}{12}\frac{\mu^2 l^2 (2\pi f)^2}{R_0^2} = .0151 .$$

$$R' = R_0(1 + .0151) = 1.0151 R_0 .$$

That is, neglecting the negative term in Gray's equation the increase is calculated as 1.5% increase over ohmic. But allowing for all chances of discrepancy this error is allowable. Therefore there is no inconsistency.

Attention should now be called to the frequency curves, page (38). With constant current density and variable frequency there is an increase in (R) and (X) and consequently in (Z), similar to that due to increasing current density. The variation here is not evidently according to the square root of frequency, (f). The resistance and impedance, however, approach ohmic resistance as they must, according to the theory. The reactance does not so evidently approach zero at zero frequency. But assuming that it does approach zero along a dotted line, it may be seen that the value of inductance dropped gradually as the frequency increased, since the reactance curve would otherwise be a straight line. This is due to the decreasing value of internal flux with increasing frequency. According to this theory in the limit, of infinite frequency, there would be no internal flux and no internal inductance.

During the study of these curves the theoretical values

for the resistance curve on page (40) were calculated according to Gray's formula,

$$R' = R_o \left(1 + \frac{1}{12}\frac{\mu^2 l^2 (2\pi f)^2}{R^2} - \frac{1}{180}\frac{\mu^4 l^4 (2\pi f)^4}{R^4} + \cdots\right)$$

and plotted on the same sheet. The difference between this (See final note) theoretical curve and the experimental one is probably due to hysteresis in the steel. Such a loss would in the method used be charged against resistance in the I R loss.

To speculate upon the probability of this effect of hysteresis let another example be taken. Estimating the depth of penetration, let the value of (B) necessary to give the required hysteresis loss be estimated. Then let the value of (B) throughout the depth of penetration be estimated from considerations of magneto-motive-force due to the current, the permeability as calculated from the reactance, and the length of magnetic path. Then let these values of (B) arrived at by the different processes be compared.

$$W = \eta \mathcal{B}^{1.6}$$

$\eta = 2 \cdot 10^{-3}$ $\mathcal{B}$ = lines per sq. cm. W = ergs/cu. cm./~

$W = 2 \cdot \mathcal{B}^{1.6} 10^{-10}$ Joules/cycle/cu. cm.

$W = 120 \cdot \mathcal{B}^{1.6} 10^{-10}$ watts/cu. cm. at 60

At $I = 6.$ amp. on curve page (40),

$R_{eff.} - R_{th.} = .233 - .18 = .053\ \omega.$

Equivalent $I^2R = 6^2 \times .053 = 1.91\ w$

Length of cable $= 64' = 1950$ cm.

Figuring penetration depth from theoretical curve

$R_o/R_{eff} = .137/.18 = .76$ part of area acting.

Area, 3/16" D, $= 0.178$ sq. cm.

$0.76 \cdot .178 \cdot 1950. = 264.$ cu. cm. Volume.

$$\frac{1.91\ w}{264.\ cu.cm.} = 0.00724 \text{ watts per cu. cm.}$$

$$0.00724 = 120 \cdot \mathcal{B}^{1.6} \cdot 10^{-10}$$

$$\mathcal{B} = 4000. \text{ lines per sq. cm.}$$

To calculate(B) from m.m.f. and (μ);

The average length of magnetic circuit is

$$l = \pi \cdot 3/16'' \cdot 0.75$$

$$= 1.125 \text{ cm.}$$

$$\mathcal{B} = 0.4\,\pi I l \mu.$$

$I = 6.$; $\mu = 414.$ from reactance calculation (See final note.)

$$\mathcal{B} = 0.4\,\pi 6 \cdot 1.125 \cdot 414.$$

$$= 3400. \text{ lines per square cm.}$$

Before speculating upon the possibility of further investigation it is well to discuss some of the points which are of practical interest. The effect of stranding the conductor is this; the per cent increase of resistance and of reactance is much less marked in the case of the stranded cable than in the solid steel wire although the larger conductor, other things being equal, would show the more marked effect due to unequal alternating current distribution.

Dismissing the solid steel conductor from consideration and assuming the extra expense of stranding not of importance, it is evident from the curves that except for very high voltages of transmission, steel would be impractical for long distance transmission in general. For short branch lines, however, stranded steel is worthy of consideration. With this assump-

tion the tables of line constants pages (45) and (46) have been prepared.

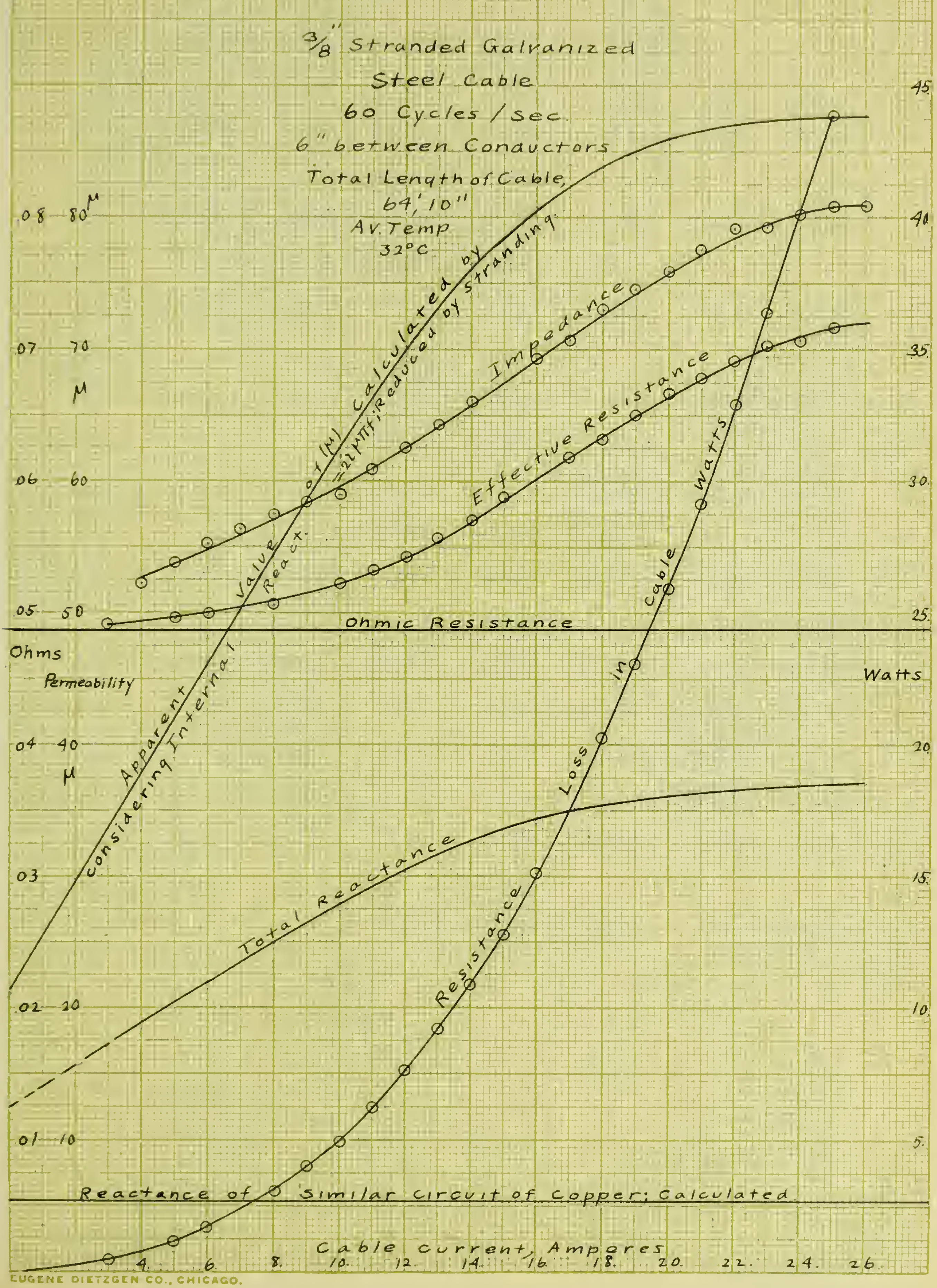
The difficulty in the fore-going theoretical interpretation of the results of the tests lies in the uncertainty of the effect of hysteresis upon the distribution of alternating current in a conductor of magnetic material. The apparent resistance can be measured. But there are involved three unknown, depth of penetration, magnetic permeability and hysteresis loss. Assuming that hysteresis merely causes an energy loss and does not affect depth of penetration, any one of these three factors, determines the other two. But this assumption on the effect of hysteresis is not a safe one. If by experimental tests and mathematics the relation of hysteresis to alternating current distribution in a magnetic material is determined, the two factors in the effective resistance as measured, ohmic resistance and hysteresis may be separated. Not only would this make possible an understanding of the distribution of alternating current and alternating magnetic flux in a ferric conductor but there would be a possible method of investigating the permeability of iron at different flux densities. This would be an interesting subject for future investigation.

Another suggestion to any student who takes up this problem is that he use a wider range of frequencies than was used in the above tests.

NOTE:---- Further explanation of the theoretical effective resistance curve, page (40) is advisable. In solving for these values of this curve the penetration depths were calculated from the experimental values of effective resistance. This allowed an approximation of the values of (μ) which entered into the known values of internal reactance. These values of (μ) are too large but when substituted in Gray's formula for effective resistance they give values of resistance which when plotted lie along the highest position possible for the correct theoretical curve. Although this introduces error into all of the hysteresis calculations, the latter indicate the possible truth of the assumption made.

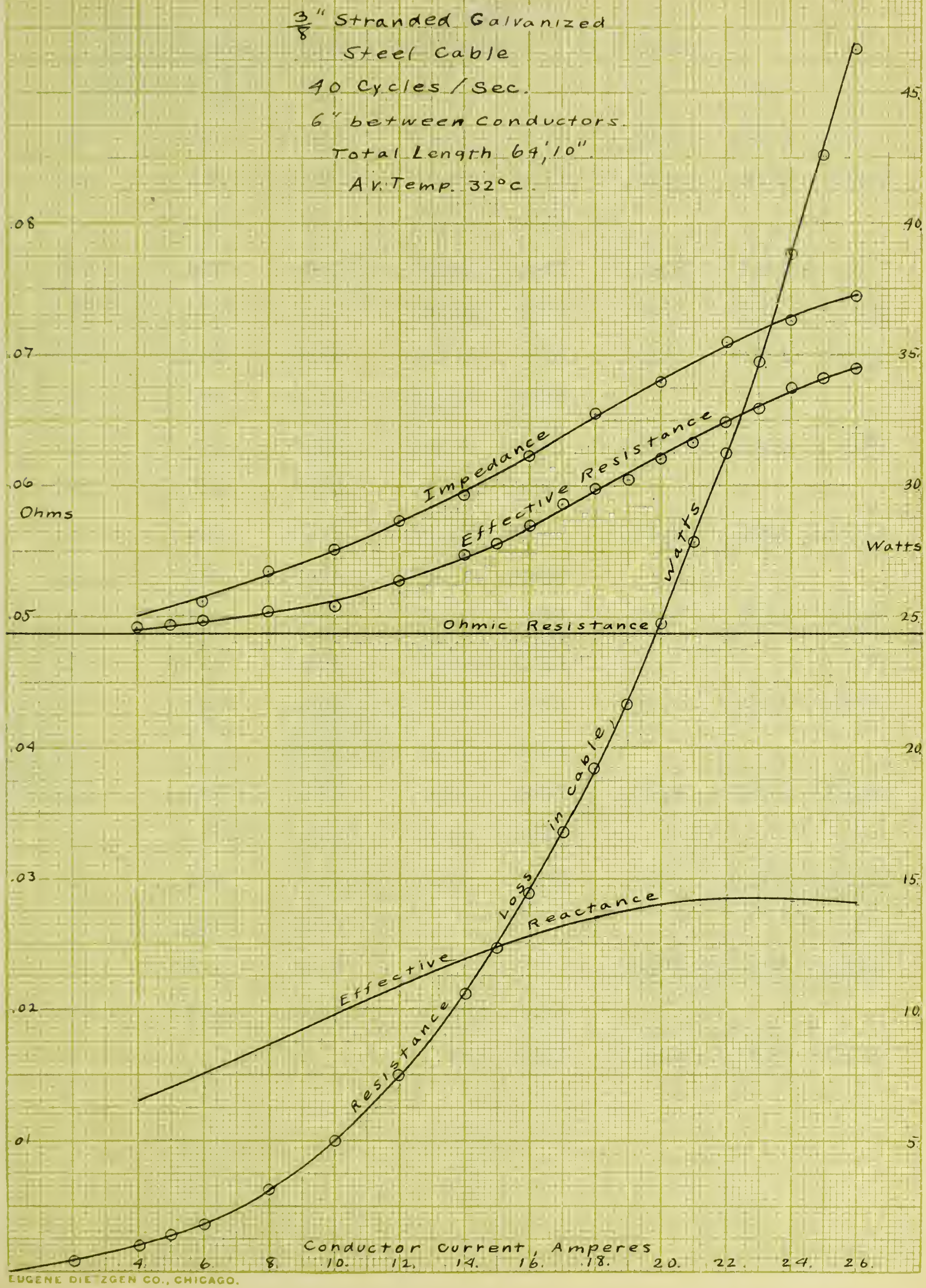
3/8 INCH STRANDED STEEL CABLE. 60 CYCLES.

I	E	Z	W	R	Z_c	R_c	X
26.	2.1	.0808			.081	.0720	.0373
25.	2.02	.0808	44.63	.0716			
24.	1.925	.0802	40.90	.0709	.0802	.0710	.0373
23.	1.825	.0794	37.08	.0702			
22.	1.740	.0791	33.48	.0692	.0785	.0695	.0366
21.	1.630	.0776	29.80	.0680			
20.	1.520	.0760	26.44	.0667	.0760	.0666	.0365
19.	1.415	.0745	23.56	.0652			
18.	1.313	.0730	20.65	.0632	.0730	.0632	.0365
17.	1.200	.0706	17.88	.0620			
16.	1.105	.0691			.0695	.0602	.0361
15.	1.015	.0677	13.13	.0588			
14.	0.930	.0664	11.20	.0572	.0660	.0572	.0330
13.	0.835	.0642	9.47	.0558			
12.	0.752	.0626	7.89	.0544	.0624	.0545	.0305
11.	0.670	.0609	6.40	.0533			
10.	0.590	.0590	5.21	.0523	.0594	.0523	.0281
9.	0.525	.0584					
8.	0.460	.0575	3.250	.0507	.0568	.0510	.0251
7.	0.395	.0564					
6.	0.330	.0550	1.793	.0500	.0547	.0500	.0221
5.	0.266	.0532	1.242	.0496			
4.	0.209	.0524			.0530	.0494	.0192
3.	0.144	.0480	.443	.0491			


3/8" Stranded Galvanized
Steel Cable
60 Cycles / sec
6" between Conductors
Total Length of Cable,
64' 10"
Av. Temp
32°C
Apparent Value of (μ) Calculated by
considering Internal React. = 2 μπf; Reduced by Stranding
Impedance
Effective Resistance
Ohmic Resistance
Loss in Cable Watts
Total Reactance
Resistance
Reactance of Similar circuit of Copper; Calculated
Cable current, Amperes
Ohms
Permeability
μ
Watts
.08 80
.07 70
.06 60
.05 50
.04 40
.03
.02 20
.01 10
45
40
35
30
25
20
15
10
5
4. 6. 8. 10. 12. 14. 16. 18. 20. 22. 24. 26

3/8 INCH STRANDED STEEL CABLE. 40 CYCLES.

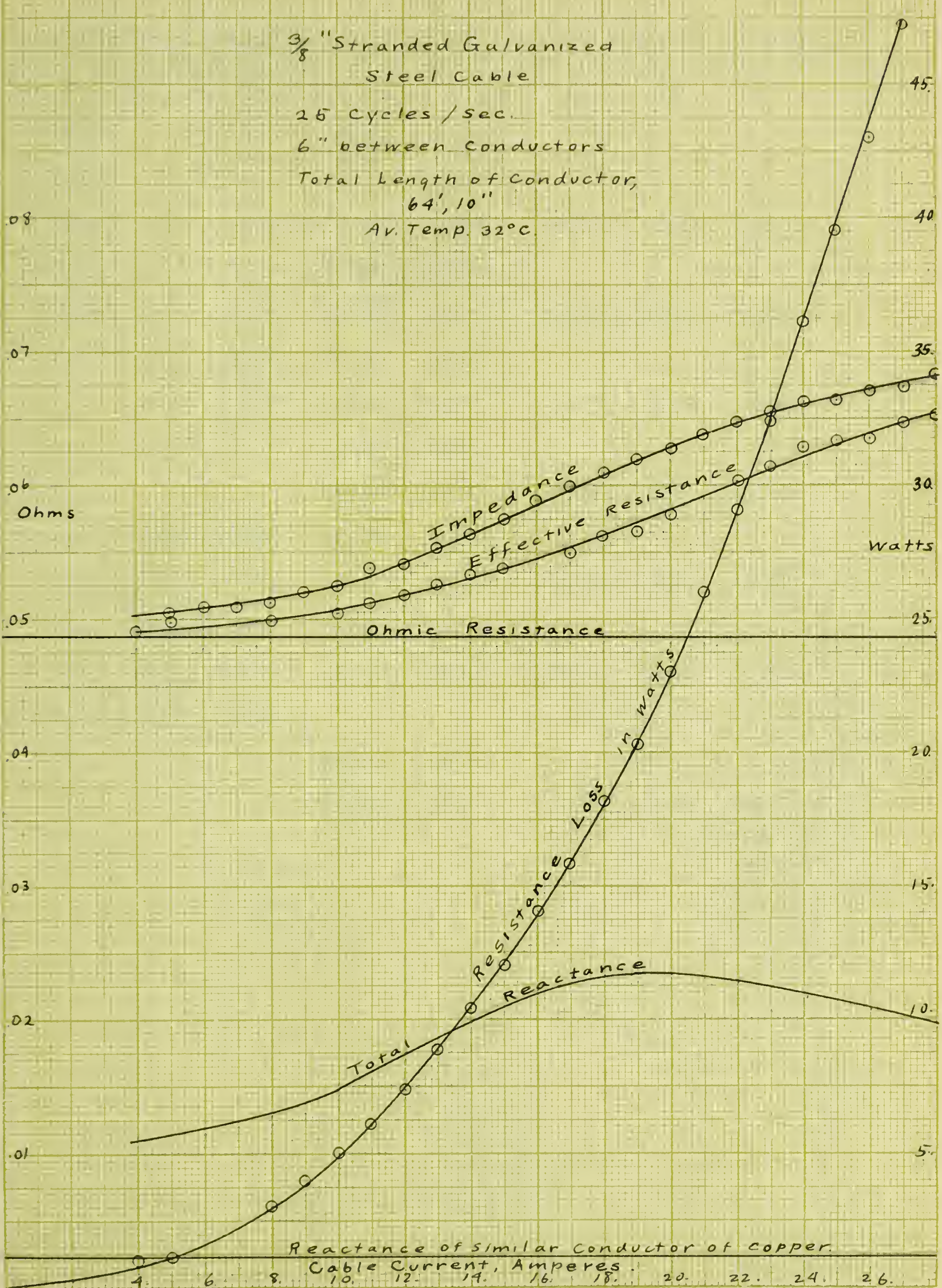
I	E	Z	W	R	Z_c	R_c	X
26.	1.922	.0745	46.55	.0689	.0745	.069	.0281
25.			42.60	.0682			
24.	1.725	.0725	38.84	.0674	.0730	.0672	.0285
23.			34.80	.0659			
22.	1.545	.0710	31.35	.0649	.0710	.0648	.0290
21.			27.93	.0633			
20.	1.346	.0679	24.80	.0682	.0682	.0622	.0281
19.			21.82	.0605			
18.	1.169	.0655	19.35	.0598	.0655	.0595	.0274
17.			16.93	.0586			
16.	.988	.0622	14.58	.0570	.0625	.0570	.0257
15.			12.40	.0556			
14.	.824	.0592	10.73	.0547	.0595	.0545	.0239
12.	.683	.0574	7.606	.0528	.0570	.0525	.0221
10.	.546	.0551	5.083	.0508	.0550	.0512	.0200
8.	.424	.0534	3.225	.0504	.0532	.0503	.0173
6.	.304	.0512	1.784	.0498	.0517	.0497	.0141
5.			1.240	.0496			
4.			.787	.0491	.0510	.0492	.0134



3/8" Stranded Galvanized
Steel Cable
40 Cycles / Sec.
6" between conductors.
Total Length 64' 10"
Av. Temp. 32°c.
Impedance
Effective Resistance
Ohmic Resistance
Watts loss in cable
Effective Reactance
Effective Resistance
Ohms
Watts
.08
.07
.06
.05
.04
.03
.02
.01
45.
40.
35.
30.
25.
20.
15.
10.
5.
Conductor current, Amperes
4.
6.
8.
10.
12.
14.
16.
18.
20.
22.
24.
26.

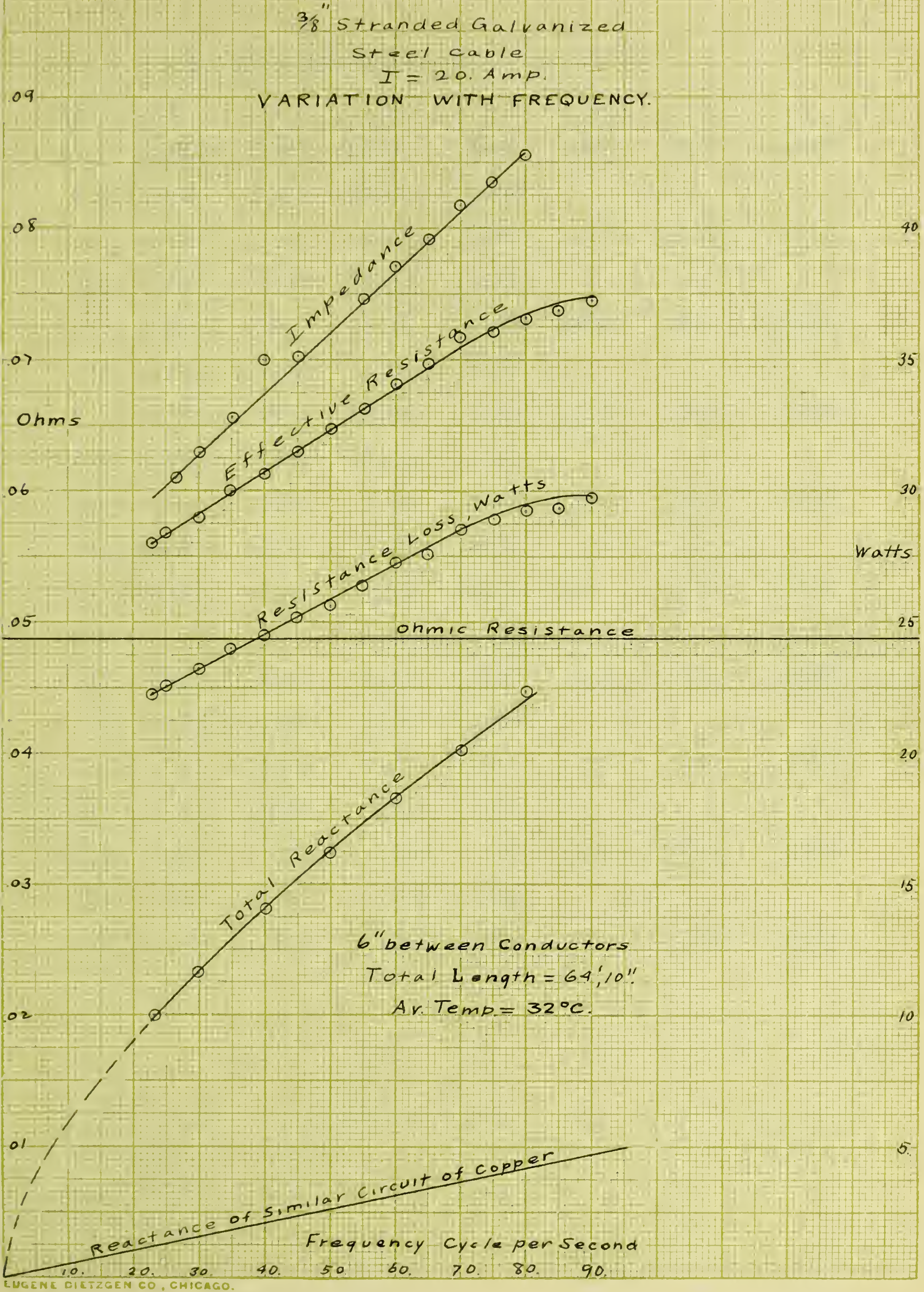
3/8 INCH STRANDED STEEL CABLE. 25 CYCLES.

I	E	Z	W	R	Z_c	R_c	X
28.	1.90	.0679	51.0	.0651	.0681	.0651	.02
26.	1.741	.0670	42.96	.0635	.0674	.0642	.0206
25.	1.660	.0664	39.50	.0633			
24.	1.585	.0661	36.10	.0627	.0662	.0625	.0217
22.	1.423	.0647	29.10	.0602	.0646	.0605	.0227
20.	1.252	.0626	23.02	.0576	.0627	.0581	.0234
19.	1.174	.0618	20.38	.0565			
18.	1.095	.0608	18.15	.0561	.0607	.0560	.0233
17.	1.017	.0598	15.84	.0549			
16.	.940	.0588	14.02	.0547	.0585	.0542	.0220
15.	.861	.0574	12.10	.0538			
14.	.788	.0563	10.42	.0532	.0563	.0527	.0197
13.	.716	.0551	8.87	.0525			
12.	.648	.0540	7.47	.0518	.0544	.0515	.0176
11.	.592	.0538	6.20	.0512			
10.	.523	.0523	5.04	.0504	.0527	.0505	.0152
9.	.468	.0520	4.09	.0500			
8.	.409	.0511	3.18	.0497	.0515	.0500	.01224
7.	.356	.0509					
6.	.310	.0508			.0508	.0495	.0114
5.	.255	.0505	1.244	.0497			
4.			.787	.0491	.0505	.0492	.0114


3/8 "Stranded Galvanized
Steel Cable
25 Cycles / Sec.
6" between Conductors
Total Length of Conductor,
64', 10"
Av. Temp. 32°C
Ohms
Watts
Impedance
Effective Resistance
Ohmic Resistance
Resistance Loss in Watts
Total Reactance
Reactance of Similar Conductor of Copper.
Cable Current, Amperes.
.08
.07
.06
.05
.04
.03
.02
.01
45.
40.
35.
30.
25.
20.
15.
10.
5.
4.
6
8.
10.
12.
14.
16.
18.
20.
22.
24.
26.

3/8 INCH STRANDED STEEL CABLE. I = 20 VARIABLE FREQUENCY

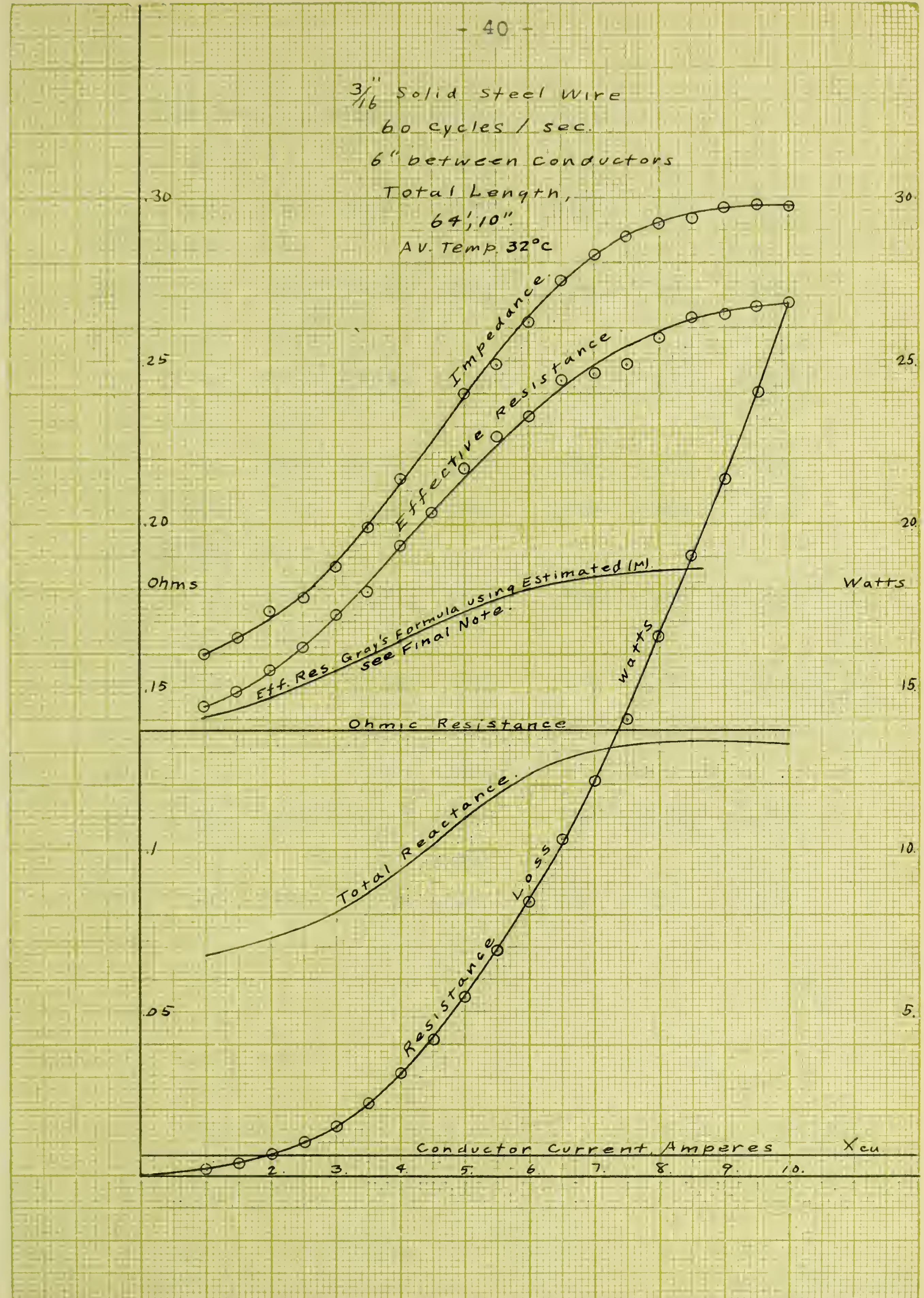
F	E	Z	W	R	Z_c	R_c	X
90.			29.8	.0744			
85.			29.5	.0737			
80.	1.718	.0859	29.25	.0732	.0864	.0740	.0447
75.	1.668	.0834	28.86	.0722			
70.	1.632	.0816	28.66	.0717	.0815	.0710	.0401
65.	1.586	.0793	27.65	.0697			
60.	1.542	.0771	27.25	.0681	.0770	.0678	.0365
55.	1.492	.0746	26.43	.0661			
50.			25.62	.0646	.0722	.0645	.0324
45.	1.404	.0702	25.20	.0630			
40.	1.400	.0700	24.50	.0612	.0675	.0614	.0281
35.	1.312	.0656	24.00	.0600			
30.	1.260	.0630	23.18	.0580	.0630	.0583	.0239
26.5	1.218	.0609					
25.			22.66	.0568			
23.			22.36	.0560	.0595	.0560	.02



3/8" Stranded Galvanized
Steel Cable
I = 20. Amp.
VARIATION WITH FREQUENCY.
.09
.08
.07
.06
.05
.04
.03
.02
.01
Ohms
40
35
30
25
20
15
10
5.
Watts
Impedance
Effective Resistance
Resistance Loss, Watts
Ohmic Resistance
Total Reactance
6" between Conductors
Total Length = 64' 10".
Av. Temp = 32°C.
Reactance of Similar Circuit of Copper
Frequency Cycle per Second
10. 20. 30. 40. 50. 60. 70. 80. 90.

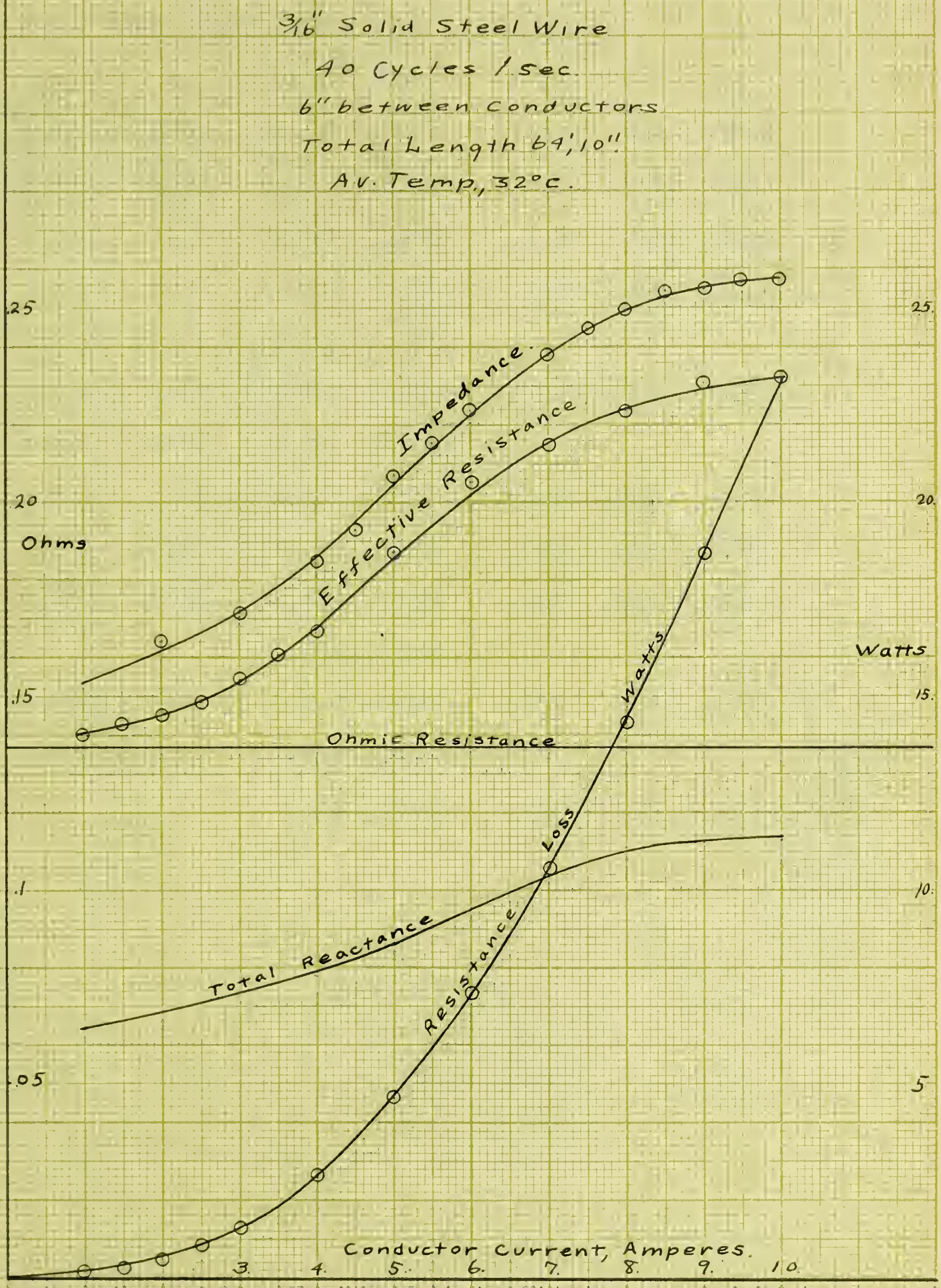
3/16 INCH SOLID STEEL WIRE. 60 CYCLES

I	E	Z	W	R	Z_c	R_c	X
10.	2.971	.2971	26.8	.2680	.297	.267	.1300
9.5	2.827	.2977	24.05	.2665			
9.	2.670	.2968	21.35	.2640	.297	.265	.1338
8.5	2.500	.2940	19.03	.2635			
8.	2.330	.2913	16.45	.2570	.292	.26	.1326
7.5	2.155	.2875	14.01	.2490			
7.	1.975	.2820	12.05	.2460	.282	.249	.1322
6.5	1.785	.2750	10.30	.2440			
6.0	1.570	.2620	8.40	.2333	.264	.233	.1245
5.5	1.370	.2490	6.86	.2270			
5.	1.201	.2400	5.43	.2170	.24	.213	.1100
4.5			4.115	.2030			
4.0	.857	.2140	3.10	.1937	.213	.191	.0944
3.5	.698	.1990	2.20	.1795			
3.0	.561	.1870	1.55	.1723	.187	.170	.0781
2.5	.446	.1780	1.012	.1620			
2.0	.345	.1730	.624	.1550	.171	.154	.0748
1.5	.248	.1650	.335	.1490			
1.0	.160	.1600	.144	.1440	.16	.144	.0693


3/16" Solid Steel Wire
60 cycles / sec.
6" between Conductors
Total Length,
64', 10".
AV. Temp. 32°C
Impedance
Effective Resistance
Eff. Res. Gray's Formula using Estimated (M).
see Final Note.
Ohmic Resistance
Total Reactance.
Resistance Loss Watts
Ohms
Watts
.30
.25
.20
.15
.1
.05
30.
25.
20.
15.
10.
5.
Conductor Current, Amperes
X_{cu}
2.
3.
4.
5.
6.
7.
8.
9.
10.

3/16 INCH SOLID STEEL WIRE. 40 CYCLES.

I	E	Z	W	R	Z_c	R_c	X
10.	2.580	.258	23.17	.2317	.258	.232	.1126
9.5	2.438	.257					
9.0	2.300	.2555	18.70	.2310	.256	.23	.1126
8.5	2.142	.2545					
8.0	2.000	.2500	14.34	.2240	.250	.224	.1108
7.5	1.840	.2453					
7.0	1.670	.2386	10.54	.2150	.239	.214	.1063
6.	1.345	.2240	7.40	.2055	.223	.202	.0949
5.5	1.186	.2160					
5.0	1.040	.2080	4.701	.1880	.205	.186	.0860
4.5	.870	.1933					
4.0	.739	.1847	2.680	.1675	.187	.170	.0781
3.0	.517	.1720	1.397	.1552	.171	.155	.0721
2.5			.932	.1490			
2.0	.330	.1650	.584	.1460	.161	.145	.0707
1.5			.322	.1431			
1.0	.143	.1430	.1404	.1404	.154	.14	.0648



3/16" Solid Steel Wire
40 Cycles / sec.
6" between Conductors
Total Length 64', 10".
Av. Temp., 32°c.
Ohms
Watts
Impedance.
Effective Resistance
Ohmic Resistance
Watts
Loss
Resistance
Total Reactance
Conductor Current, Amperes.
.25
.20
.15
.1
.05
25.
20.
15.
10.
5
3.
4.
5.
6.
7.
8.
9.
10.

3/16 INCH SOLID STEEL WIRE. 25 CYCLES.

I	E	Z	W	R	Z	R	X
9.5	2.016	.2122	18.06	.2003	.213	.200	.0742
9.0	1.913	.2126	16.10	.1988	.213	.199	.0768
8.0	1.692	.2115	12.37	.1933	.211	.195	.0800
7.0	1.448	.2070	9.25	.1888	.206	.189	.0842
6.5	1.316	.2025	7.83	.1850			
6.0	1.188	.1980	6.53	.1814	.198	.180	.0824
5.5	1.056	.1920	5.25	.1742			
5.0	.928	.1855	4.190	.1675	.185	.168	.0781
4.0	.679	.1700	2.497	.1560	.170	.156	.0671
3.0	.466	.1553	1.326	.1473	.157	.147	.0557
2.5			.900	.1440			
2.0	.3005	.1500	.568	.1420	.150	.142	.0480
1.5	.2216	.1477	.315	.1400			
1.0			.139	.1390	.146	.140	.0412

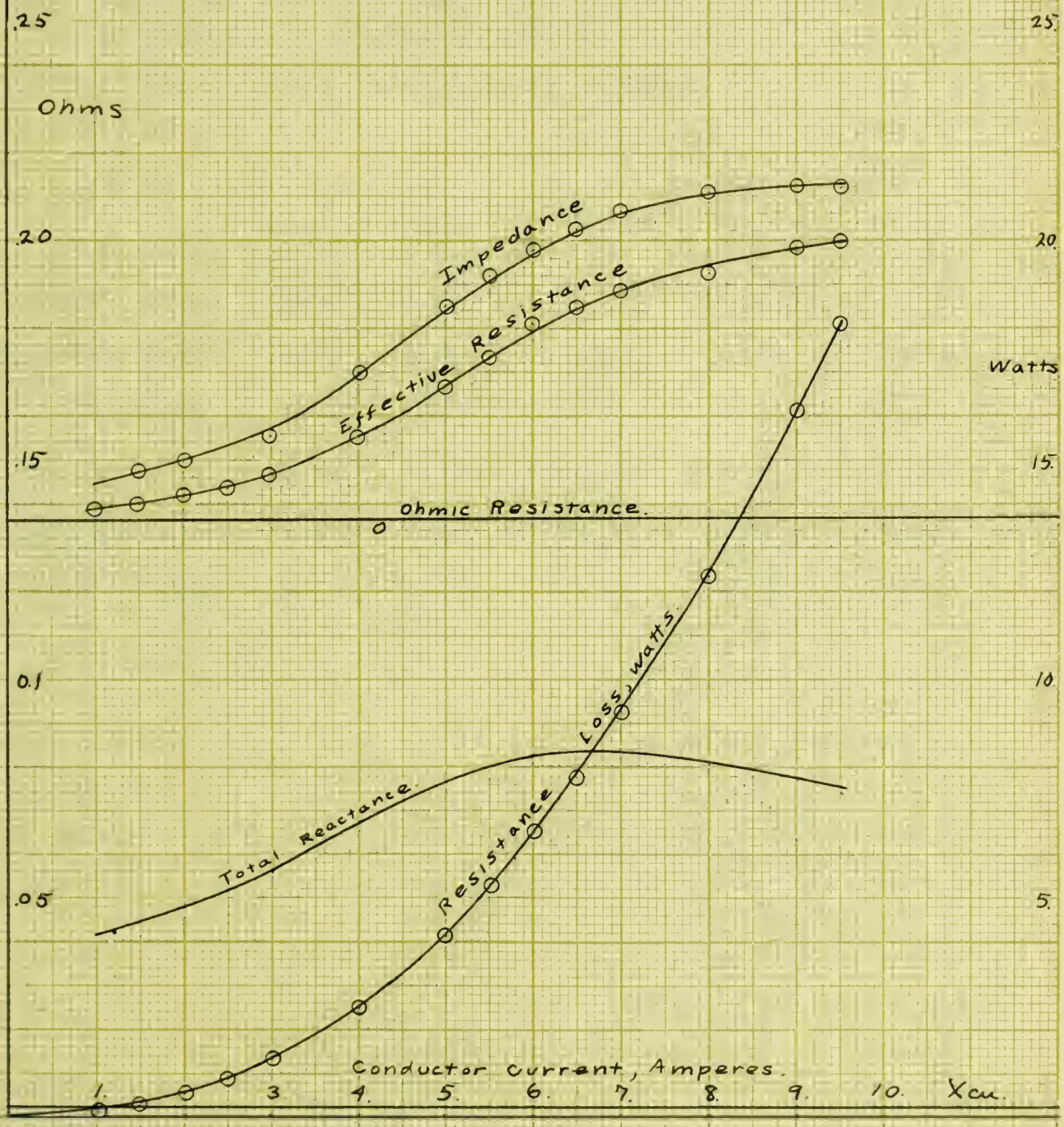
3/16" Solid Steel Wire

25 cycles/sec.

6" between Conductors

Total Length, 64' 10".

Av. Temp., 32° C.



Transmission Line Constants

for

3/8" Stranded Galvanized Steel Cable

60 cycle Alternating current

for

Single conductor per mile

D = spacing distance in inches

I amp.	4.	8.	12.	16.	20.	25.
R ω/mi.	4.07	4.21	4.50	4.97	5.50	5.94
X ω/mi. D=6"	2.42	2.91	3.37	3.69	3.84	3.91
X ω/mi. D-12"	2.59	3.07	3.54	3.86	4.01	4.08
X ω/mi. D=18"	2.69	3.17	3.64	3.96	4.11	4.18
X ω/mi. D=24"	2.76	3.24	3.71	4.03	4.18	4.25
X ω/mi. D=30"	2.81	3.29	3.76	4.08	4.23	4.30
X ω/mi. D=36"	2.86	3.34	3.81	4.13	4.28	4.35

Transmission Line Constants

for

3/8" Stranded Galvanized Steel Cable

25 cycle Alternating Current

for

Single conductor per mile

D = spacing distance in inches

I amp.	4	8	12	16	20	25
R ω/mi.	4.06	4.11	4.25	4.47	4.79	5.24
X ω/mi. D=6"	1.26	1.42	1.81	2.17	2.28	2.10
X ω/mi. D=12"	1.33	1.49	1.88	2.24	2.34	2.17
X ω/mi. D=18"	1.37	1.53	1.92	2.28	2.38	2.21
X ω/mi. D=24"	1.40	1.56	1.95	2.31	2.41	2.24
X ω/mi. D=30"	1.42	1.59	1.97	2.33	2.44	2.26
X ω/mi. D=36"	1.44	1.60	1.99	2.35	2.46	2.28